Item Popularity

Prediction

 $\hat{x}_{ui} = \frac{\#\text{interactions with item }i}{\#\text{interactions}}$

Power-Law Nature of #Interactions



Frequency Distribution of Movielens-20M Ratings

Pros & Cons

- + simple & efficient
- + no parameters to train
- + solves cold-start problem
- + works well due to power-law nature of interactions
- no tailoring of recommendations to users' specific preferences

Matrix Factorization (MF)

Prediction

$$\hat{x}_{u,i} = \langle \mathbf{x}_u, \mathbf{x}_i \rangle + \mu_u + \mu_i + \mu$$

Training Objective Squared Loss or BPR Loss



Pros & Cons

- + Fast & efficient predictions tailored to user's tastes
- + Well-established model with lots of extensions (ALS, time incorporation, online algs.,
 - \mathbb{R} Non-neg. MF, ...)
- + Interpretability
- + Usable with MIPS
- Fails to learn user-user and item-item similarities
- Prediction is only a bi-linear

Image Source: "Matrix factorization techniques for recommende[®] systems" Y Koren, R Bell, C Volinsky

 \hat{r}_{ui}

Collaborative Metric Learning (CML) $(1/3)^4$

Embedding of users and items in a *metric space* $(U \cup I, d)$ with distance metric d(u, i).



⁴ "Collaborative Metric Learning", C K Hsieh, L Yang, Y Cui, T Y Lin, S Belongie, D Estrin

Collaborative Metric Learning (CML) (2/3)

Profits from the phenomenon of *similarity propagation*, because distance metric d respects the *triangle inequality*:

$$\forall u, i, j: \quad d(i, j) \le d(u, i) + d(u, j)$$

 \rightarrow Similarity of (u,i) and (u,j) is propagated to (i,j).



 \rightarrow User-user and item-item similarities automatically learned.

Collaborative Metric Learning (CML) (3/3)

Prediction

$$\hat{x}_{ui} = -d(\mathbf{x}_u, \mathbf{x}_i)$$

Training Objective WARP Loss with Cov. Reg.

$$\mathcal{L}(\boldsymbol{\theta}) = \mathcal{L}_m(\boldsymbol{\theta}) + \lambda \Omega(\boldsymbol{\theta}) \quad \text{s.t. } \|\mathbf{x}_*\| \leq 1.$$

$$\mathcal{L}_m(\boldsymbol{\theta}) = \sum_{(i,j)\in\mathcal{S}} \sum_{(u,k)\notin\mathcal{S}} w_{ij} \left[m + d(\mathbf{x}_u, \mathbf{x}_i)^2 - d(\mathbf{x}_u, \mathbf{x}_j)^2 \right]_+,$$

Pros & Cons

- + Benefits from *similarity propagation* → user-user &
 item-item similarities
 automatically learned
- + Interpretability
- + LSH possible
- Metric space geometry must suit the latent geometry

Hyperbolic Recommender Systems $(1/2)^6$

Harness *hyperbolic* metric space to represent the relevances.

<figure>

Poincaré Ball



Motivation:⁵



 ⁵ "Hyperbolic geometry of complex networks", D Krioukov, F Papadopoulos, M Kitsak, A Vahdat, M Boguná
 ⁶ "Hyperbolic recommender systems", T D Q Vinh, Y Tay, S Zhang, G Cong, X L Li

[&]quot;Scalable Hyperbolic Recommender Systems" B. P. Chamberlain, S. R. Hardwick, D. R. Wardrope, F. Dzogang, F. Daolio, S. Vargas

Hyperbolic Recommender Systems (2/2)

Prediction

$$\hat{x}_{ui} = -\alpha d(\mathbf{x}_u, \mathbf{x}_i), \quad \alpha > 0.$$



Pros & Cons

- + Similarity propagation
- + Interpretability
- + LSH possible
- Prediction power depends on suitability of geometry

Training Objective (BPR)

$$\mathcal{L}(\boldsymbol{\theta}) = \sum_{(u,i,j)\in\mathcal{D}} -\log\left(\sigma\left(\alpha\left(d(\mathbf{x}_u,\mathbf{x}_j) - d(\mathbf{x}_u,\mathbf{x}_i)\right)\right)\right),$$

Neural Collaborative Filtering (NCF)³

Prediction



Pros & Cons

- + Non-linear prediction
- + Robust joint-model of simple GMF and MLP
- Lacks interpretability
- Expensive prediction
- Cannot be used with
 LSH or MIPS

Training Objective Binary Cross-Entropy with neg. sampling

$$\mathcal{L}(\boldsymbol{\theta}) = -\sum_{(u,i)\in\mathcal{D}} \left[x_{ui} \log(\hat{x}_{ui}) + (1 - x_{ui}) \log(1 - \hat{x}_{ui}) \right]$$

 $^{^3}$ "Neural collaborative filtering", X He, L Liao, H Zhang, L Nie, X Hu, T S Chua,

Autoencoders for Collaborative Filtering²

```
Prediction \hat{x}_{ui} = h(\mathbf{x}_u, \boldsymbol{\theta})
```



Training Objective Reconstruction/Prediction Error or ELBO

Pros & Cons

- + Non-Linear ranking
- + No negative sampling needed
- Lack of interpretability
- No automatic discovery of user-user item-item similarities
- Predictions over *all* users or items $\Theta(\min |U|, |I|)$
- LSH or MIPS not possible

Major Challenge: Sparsity of inputs/gradients

- Item-based RS
- Dense re-feeding: $h(h(\mathbf{x})) = h(\mathbf{x})$

² "Autorec: Autoencoders meet collaborative filtering." S Sedhain, A K M, S Sanner, L Xie "Collaborative filtering with stacked denoising autoencoders and sparse inputs.", F Strub, M Jeremie

[&]quot;Training deep autoencoders for collaborative filtering", O Kuchaiev, B Ginsburg

[&]quot;Variational autoencoders for collaborative filtering", D Liang, R G Krishnan, M D Hoffman, T Jebara